Algebraic and approximation methods for data analysis and machine learning:

Approximation theory has a long history of developments, a significant part of it being based on the algebra of polynomials. In this context, strong theoretical results describe important mechanisms underpinning polynomial approximation. In relation with these mathematical developments, our main tool of interest is the so called Christoffel function. This function is still the object of theoretical investigations from the mathematical community, but a descent amount of knowledge is available regarding its properties.

We begin with the observation that available properties of the Christoffel function have not yet been exploited for machine learning purposes. Our preliminary work suggests that this could be a fruitful research direction ; see a successful application to fraud detection described in the article `arXiv:1606.03858` to be presented at NIPS (an annual major and selective conference for the machine learning community, this year in Barcelona, December 2016). Pushing the idea further opens many potential research directions, some of them being largely open, at the interface between approximation theory, algebra, optimization, computation and statistics. Examples include :

- Applications in statistics and machine learning : support and density estimation from samples, manifold identification, kernel methods.
- Numerical methods, algebra and optimization : the cost of increasing dimension is, as of today, too high to provide reliable tools in high dimensions.
- Other applications : detection of affine matching, shape comparison, imaging.
- Theoretical investigations of properties of the Christoffel function, stability properties, links with approximation theory and pluripotential theory.

The advisors have very strong motivations to pursue investigations along these lines. The proposed internship will be an opportunity to push one or several of the proposed points. This proposal has three main distinguishing features : (i) it uses tools from different areas of applied mathematics, (ii) the expected output can take a variety of forms, from very practical applications to abstract theory, (iii) the topic of interest is hardly touched for the moment. Depending on the progress, the successful candidate could be proposed to pursue this research during a Ph.D.

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