A geometric algorithm for implementing Yann Brenier's "transport-collapse" scheme in 3D

- **Theme:** Geometric computing and numerical analysis
- **Laboratory:** Centre Inria Nancy Grand-Est
- **Team:** ALICE http://alice.loria.fr
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- **Context:**
  The goal of this project is to develop an efficient algorithm for implementing some non-conventional numerical solution mechanisms for Partial Derivative Equations with a strong geometric component [1],[2]. Such PDEs are the models of some physics subject to conservation laws. The goal is then to derive numerical solution mechanisms that satisfy these conservation laws. Yann Brenier created the theory that expresses this point of view and invented several original methods, leading to unconventional numerical solution mechanisms. The most well known one is the fluid mechanics formulation of optimal transport that he developed with Benamou [3] (see also [4] for a gentle introduction). It is also possible to derive some semi-discrete solvers [5,6].
- **Subject:** For conservation laws, Brenier also developed a theory and an algorithm, leading to a non-conventional numerical solution mechanism, that he calls "transport-collapse" ("transport-croulement" in French) [1,2]. This algorithm was not implemented yet, because it requires some non-trivial geometric components (i.e. computing the intersection of a 3D mesh embedded in 4D). The work consists in studying and understanding the algorithm, deriving an efficient geometric algorithm to compute the intersections and implementing it. The work already realized by the team for semi-discrete optimal tranport [6] can be used as a starting point, since it involves similar computations of geometric intersections.
- **References:**

  [1] Y. Brenier, Calcul des lois de conservation scalaires par la mthode de transport-coulement, 1981, https://hal.inria.fr/inria-00076508
  [2] Y. Brenier, 2015, Mthode du transport-croulement, personnal communication
  [3] A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem JD Benamou, Y Brenier Numerische Mathematik 84 (3), 375-393
  [4] http://www.futura-sciences.com/magazines/mathematiques/infos/dossiers/d/mathematiques-maths-transport-moindre-cout-849/page/4/

[5] A multiscale approach to optimal transport. Q. Mrigot, Computer Graphics Forum 30 (5) 15831592, 2011 (also Proc SGP 2011).
[6] Bruno Levy, A numerical algorithm for L2 semi-discrete optimal transport in 3d, 2015, ESAIM Mathematical Modeling and Numerical Analysis, http://arxiv.org/abs/1409.1279

- **Required Skills:**
  Knowledge of basic computational geometry is needed. Taste for C++ implementation is appreciated.